Hong Kong Mathematics Olympiad (1995 – 96)
Final Event 6 (Group)
香港数学竞赛 (1995 – 96)
决赛项目 6 (团体)

(i) The number of eggs in a basket was a. Eggs were given out in three rounds. In the first round half of the eggs plus half an egg were given out. In the second round, half of the remaining eggs plus half an egg were given out. In the third round, again, half of the remaining eggs plus half an egg were given out. The basket then became empty. Find a.

一篮子鸡蛋的数目为 a，分三轮派发。第一轮派出一半另半枚，第二轮派出剩下的一半另半枚，第三轮又派出剩下的一半另半枚。篮子中的鸡蛋便全部派光，求 a。

$a =$

(ii) If $p-q=2$; $p-r=1$ and $b=(r-q)\left[(p-q)^2+(p-q)(p-r)+(p-r)^2\right]$. Find the value of b.

$b =$

若 $p-q=2$； $p-r=1$ 及 $b=(r-q)\left[(p-q)^2+(p-q)(p-r)+(p-r)^2\right]$，求 b 的值。

(iii) If n is a positive integer, $m^{2n}=2$ and $c=2m^{6n}-4$, find the value of c.

$c =$

若 n 是一正整数，$m^{2n}=2$ 及 $c=2m^{6n}-4$，求 c 的值。

(iv) If r, s, t, u are positive integers and $r^5=s^4$, $t^3=u^2$, $t-r=19$ and $d=u-s$, find the value of d.

$d =$

若 r, s, t, u 是正整数及 $r^5=s^4$，$t^3=u^2$，$t-r=19$ 及 $d=u-s$，求 d 的值。

Hong Kong Mathematics Olympiad (1995 – 96)

Final Event 7 (Group)

香港数学竞赛 (1995 – 96)

决赛项目 7 (团体)

(i) If the two roots of the equation $ax^2 - mx + 1996 = 0$ are primes, find the value of a.

$a =$

若方程 $ax^2 - mx + 1996 = 0$ 的两个根是质数，求 a 的值。

(ii) A six-digit figure $111aaa$ is the product of two consecutive positive integers b and $b + 1$, find the value of b.

$b =$

六位数 $111aaa$ 是两个连续正整数 b 和 $b + 1$ 之积，求 b 的值。

(iii) If p, q, r are non-zero real numbers; $p^2 + q^2 + r^2 = 1$,

$c =$

$p\left(\frac{1}{q}+\frac{1}{r}\right)+q\left(\frac{1}{r}+\frac{1}{p}\right)+r\left(\frac{1}{p}+\frac{1}{q}\right)+3=0$ and $c = p + q + r$, find the value of c.

若 p，q，r 是非零实数，$p^2 + q^2 + r^2 = 1$，

$p\left(\frac{1}{q}+\frac{1}{r}\right)+q\left(\frac{1}{r}+\frac{1}{p}\right)+r\left(\frac{1}{p}+\frac{1}{q}\right)+3=0$ 及 $c = p + q + r$,，求 c 的最大值。

(iv) If the unit digit of 7^{14} is d, find the value of d.

$d =$

若 7^{14} 之个位是 d，求 d 的值。

Hong Kong Mathematics Olympiad (1995 – 96)

Final Event 8 (Group)

香港数学竞赛 (1995 – 96)

决赛项目 8 (团体)

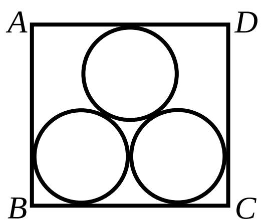
In this question, all unnamed circles are unit circles.

在本题内，所有不命名的圆皆为单位圆。

(i) If the area of the rectangle $ABCD$ is $a+4\sqrt{3}$, find the value of a.

$a =$

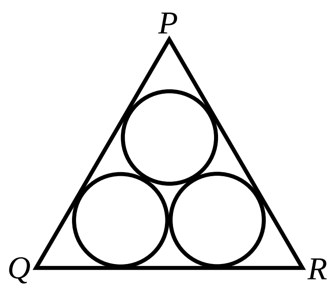
若矩形 $ABCD$ 的面积是 $a+4\sqrt{3}$，求 a 的值。



(ii) If the area of the equilateral triangle PQR is $6+b\sqrt{3}$, find the value of b.

$b =$

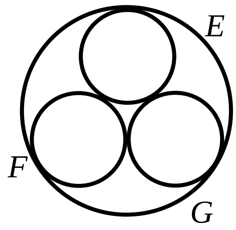
若等边三角形 PQR 的面积是 $6+b\sqrt{3}$，求 b 的值。



(iii) If the area of the circle EFG is $\dfrac{(c+4\sqrt{3})\pi}{3}$, find the value of c.

$c =$

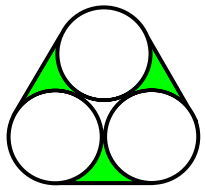
若圆 EFG 的面积是 $\frac{(c+4\sqrt{3})\pi}{3}$，求 c 的值。



(iv) If all the straight lines in the diagram below are common tangents to the two circles, and the area of the shaded part is $6+d\pi$, find the value of d.

$d =$

若下图所有直线皆为两个圆的公切线，且阴影部分的面积是 $6+d\pi$，求 d 的值。



Hong Kong Mathematics Olympiad (1995 – 96)

Final Event 9 (Group)

香港数学竞赛 (1995 – 96)

决赛项目 9 (团体)

(i) If $(1995)^a + (1996)^a + (1997)^a$ is divisible by 10, find the least possible integral value of a.

$a =$

若 $(1995)^a + (1996)^a + (1997)^a$ 能被 10 整除，求 a 的最小可能整数值。

(ii) If the expression $\left(x^2 + y^2\right)^2 \le b\left(x^4 + y^4\right)$ holds for all values of x and y, find the least possible integral value of b.

$b =$

若 $\left(x^2 + y^2\right)^2 \le b\left(x^4 + y^4\right)$ 对任意实数 x 和 y 都成立，求 b 的最小可能整数值。

(iii) If $c = 1996 \times 19971997 - 1995 \times 19961996$, find the value of c.

$c =$

若 $c = 1996 \times 19971997 - 1995 \times 19961996$，求 c 的值。

(iv) Find the sum d where

$$d = \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{60}\right) + \left(\frac{2}{3} + \frac{2}{4} + \frac{2}{5} + \cdots + \frac{2}{60}\right) + \left(\frac{3}{4} + \frac{3}{5} + \cdots + \frac{3}{60}\right) + \cdots + \left(\frac{58}{59} + \frac{58}{60}\right) + \frac{59}{60}$$

若

$$d = \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{60}\right) + \left(\frac{2}{3} + \frac{2}{4} + \frac{2}{5} + \cdots + \frac{2}{60}\right) + \left(\frac{3}{4} + \frac{3}{5} + \cdots + \frac{3}{60}\right) + \cdots + \left(\frac{58}{59} + \frac{58}{60}\right) + \frac{59}{60}$$

求 d 的值。

$d =$

Hong Kong Mathematics Olympiad (1995 – 96)
Final Event 10 (Group)
香港数学竞赛 (1995 – 96)
决赛项目 10 (团体)

(i) It is given that
已知

$$3\times4\times5\times6=19^2-1$$
$$4\times5\times6\times7=29^2-1$$
$$5\times6\times7\times8=41^2-1$$
$$6\times7\times8\times9=55^2-1$$

If $a^2=1000\times1001\times1002\times1003+1$, find the value of a.

$a=$

若 $a^2=1000\times1001\times1002\times1003+1$，求 a 的值。

(ii) Let $f(x)=x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1$. When $f(x^{10})$ is divided by $f(x)$, the remainder is b. Find the value of b.

$b=$

设 $f(x)=x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1$。当 $f(x^{10})$ 除以 $f(x)$，余数是 b。求 b 的值。

(iii) The fraction $\frac{p}{q}$ is in its simplest form. If $\frac{7}{10}<\frac{p}{q}<\frac{11}{15}$ where q is the smallest possible positive integer and $c=pq$. Find the value of c.

$c=$

分数 $\frac{p}{q}$ 已化成最简形式。若 $\frac{7}{10}<\frac{p}{q}<\frac{11}{15}$，其中 q 是最小可能正整数，且 $c=pq$，求 c 的值。

(iv) A positive integer d when divided by 7 will have 1 as its remainder; when divided by 5 will have 2 as its remainder and when divided by 3 will have 2 as its remainder. Find the least possible value of d.

$d=$

若正整数 d 除以 7，余数是 1；除以 5 余数是 2；除以 3 余数是 2。求 d 的最小可能值。